

Generic Authenticated Key Exchange in the Quantum Random Oracle Model

Kathrin Hövelmanns¹ Eike Kiltz¹
Sven Schäge¹ Dominique Unruh²

¹Horst Görtz Institute for IT Security, Ruhr University Bochum, Germany

²Institute of Computer Science, University of Tartu, Estonia

PKC 2020



Context: NIST 'competition'

Goal: Quantum-secure public-key encryption, key exchange, signatures

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Major technical problem: Probability of **decryption failure**

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Pre-quantum: DH key exchange + authentication

Post-quantum:

- DH key exchange: Broken
- Quantum Signatures: Quite costly → Can we do without them?

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Protocol 1: Relatively weak: Revealed state breaks security

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Add session-specific layer via any (passively secure) KEM

Session-specific layer + add. trick

→ Resistance against exposure of secret data

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... but the underlying scheme is assumed to be perfectly correct

→ Possibly not suitable for post-quantum ☹️

Applying FO-then-FSXY12 also results in quite a lot of hashing

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- **even for non-perfectly correct schemes**, and
- gets rid of unnecessary hashing steps

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Our proposal: 'AKE version' of Fujisaki-Okamoto

Turns passively secure PKE into post-quantum secure AKE

Kyber-Key exchange: Kyber-PKE + this work

Outline

1. The Fujisaki-Okamoto transformation
2. Two-move authenticated key exchange (AKE)
3. Our protocol: Fujisaki-Okamoto AKE
4. Open questions

Overview: The Fujisaki-Okamoto transformation

Limitations of the original work

Decryption failure?

Reminder: Property of many lattice-based encryption schemes

HHK17: Even negligible probability might affect security!

The importance of decryption failures

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Possible solutions:

1. Only build schemes with perfect correctness
 - Costly ☹️
 - What about the NIST proposals? ☹️
2. Give proofs that deal with non-perfect correctness

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Short excursion: The Quantum Random Oracle Model

Random Oracle Model (ROM)

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A can distinguish $H(x^*)$ from random

\Rightarrow Reduction learns preimage x^* (and x^* solves P)

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Question: What if A is quantum?

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Scenario: Quantum adversary interacting with non-quantum network \Rightarrow

- "Online" primitives (decryption, signing, ...) stay classical
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Example: How do we extract a particular preimage?

Extracting preimages with 'Oneway to Hiding'

"Random-until-QUERY": :

$\Pr [A \text{ distinguishes } H(x^*) \text{ from } \$] \leq \epsilon$

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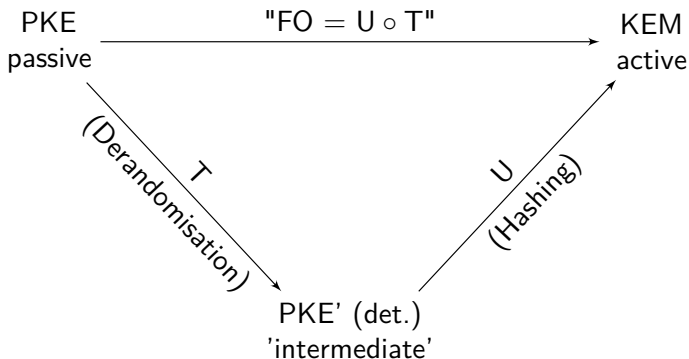
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Recent improvements :

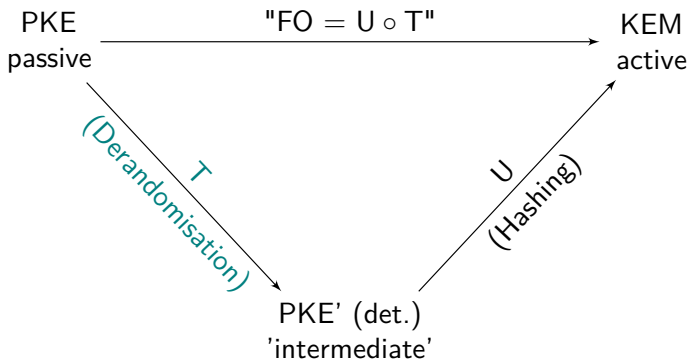
Variant	Bound
Semi-classical [AHU18]	$2\sqrt{q\epsilon}$
Double-sided [BH+19]	$2\sqrt{\epsilon}$

The FO transformation in the QRROM

Overview: Common ground of all current FO proofs



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Transformation T

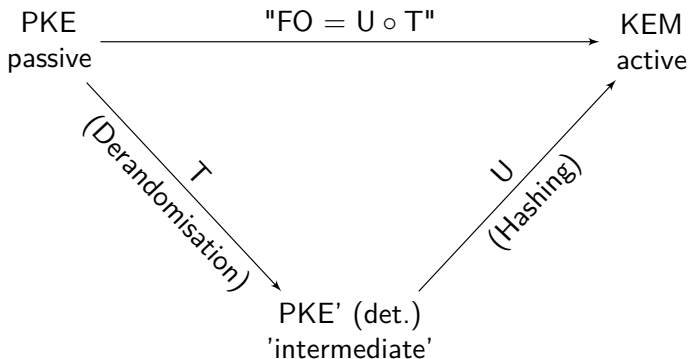
Encrypt-with-Hash construction: $\text{PKE}' := \text{T}[\text{PKE}, \text{G}]$

- Encryption: $\text{Enc}'(m) := \text{Enc}(m; \text{G}(m))$

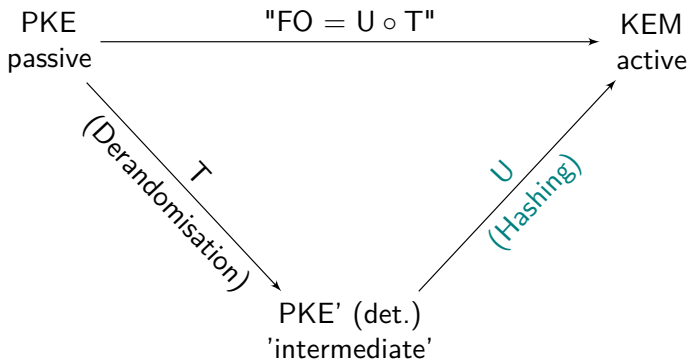
→ deterministic!

↖ Use $\text{G}(m)$ as Enc's randomness

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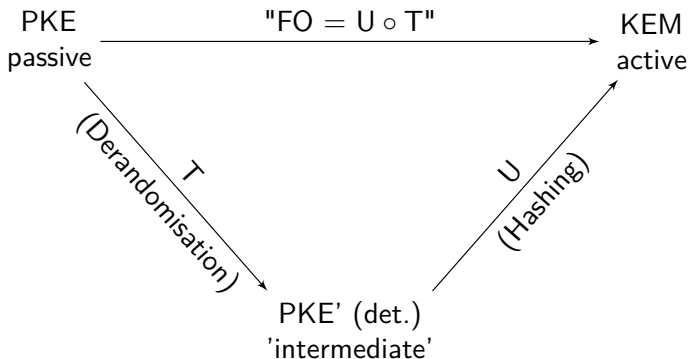
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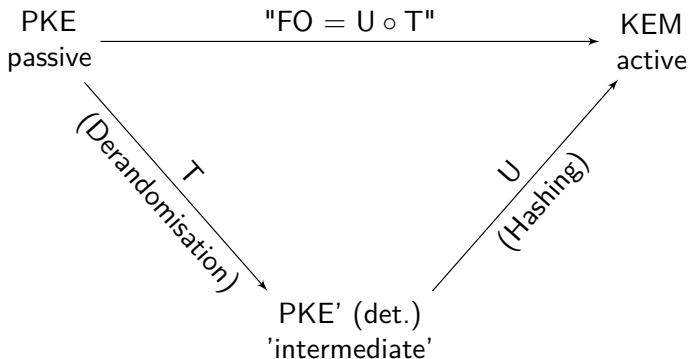
- Decapsulation:
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 3. return \perp or pseudorandom value ("implicit rejection")
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At least one step encounters quantum extraction problem

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PKE already **deterministic** \rightarrow sufficient to apply second step (U)

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'Rootless' bound:

Achieved by [new extraction technique](#) ('Measure-rewind-measure')

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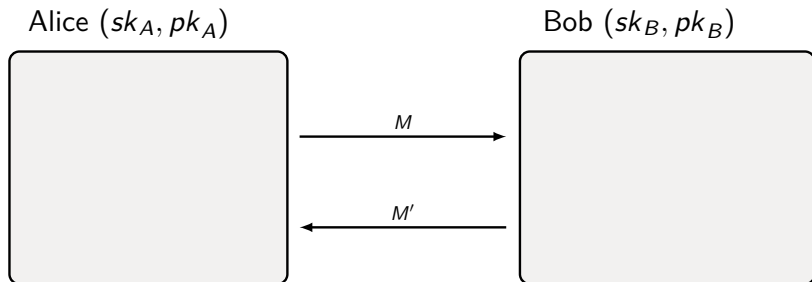
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Cave: Results for different variants (like on the U-Slide), with additional requirements

More details at <https://simons.berkeley.edu/talks/cca-encryption-qrom-i>

Authenticated key exchange

Our setting: 2-move protocols



Goal: $K = K'$ (w.o.p.), and $K \approx_c \$$

Attacking 2-move protocols

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'Tampering': Modifying the exchanged messages

Many different security models that come with subtle differences

Our security model

Two (game-based) models for two-move AKE:

1.) Key indistinguishability against active attacks

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Captures state-of-the-art attack capabilities:

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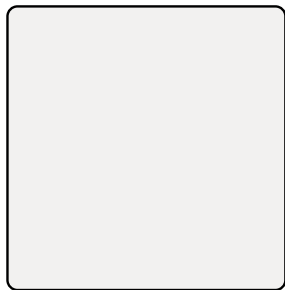
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Essentially same notion as the one used in FSXY12

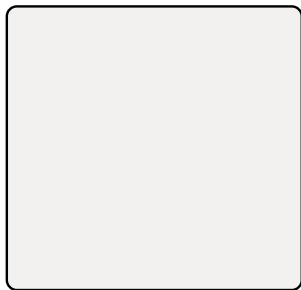
**Our protocol:
Fujisaki-Okamoto key exchange**

Our protocol

Alice (sk_A, pk_A)



Bob (sk_B, pk_B)



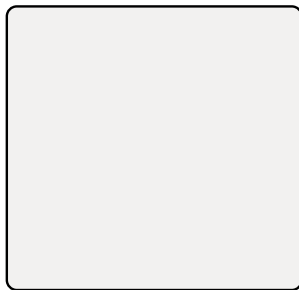
Goal: Authentication and key indistinguishability

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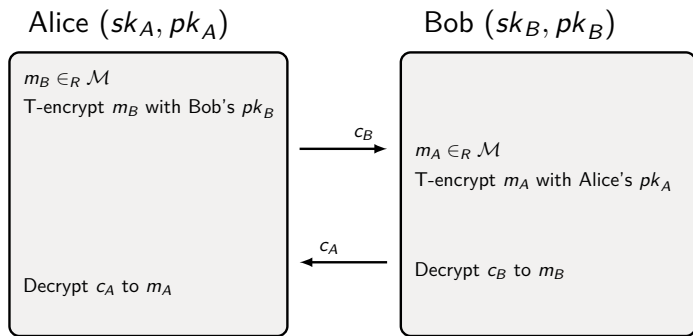
Bob (sk_B, pk_B)



Goal: Authentication and key indistinguishability

Strategy: 'Multi-user FO':

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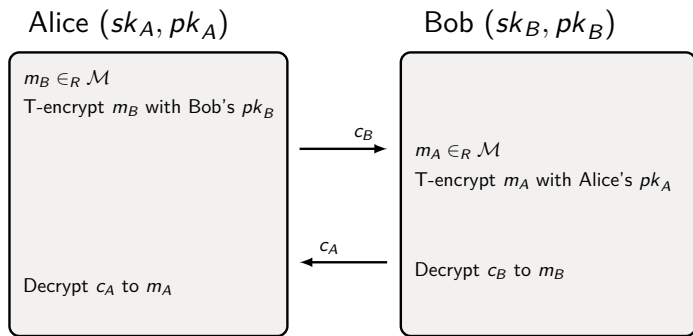


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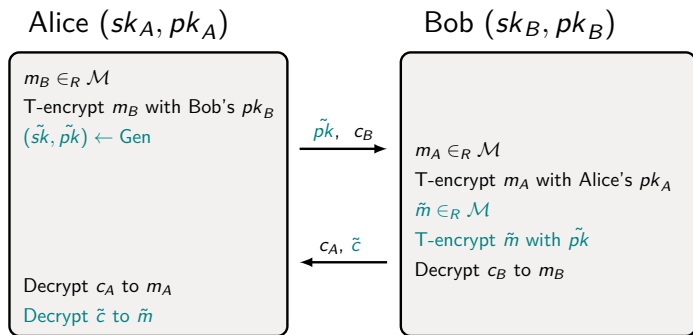
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Key computation: Multi-user variant of U-Transform

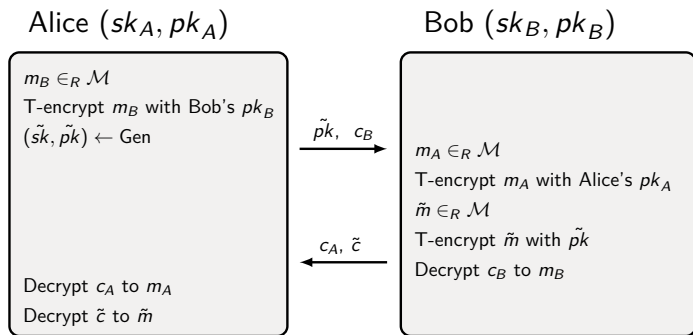
Hash whole transcript: $K := H(pk_A, pk_B, m_A, m_B, c_A, c_B)$

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Freshness: Add session-specific ('ephemeral') FO communication

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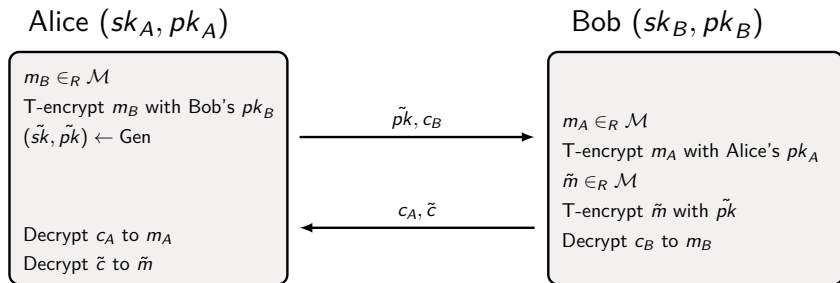


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Include 'ephemeral transcript' in hash:

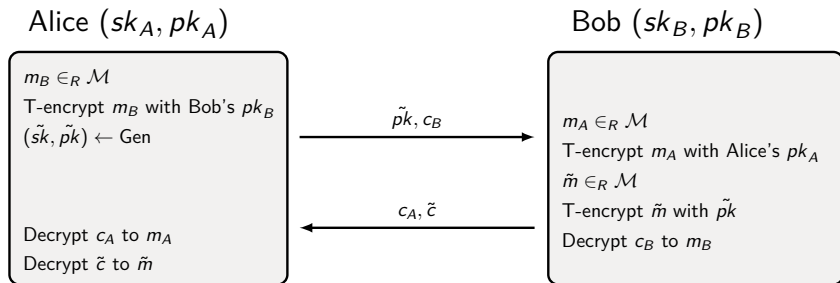
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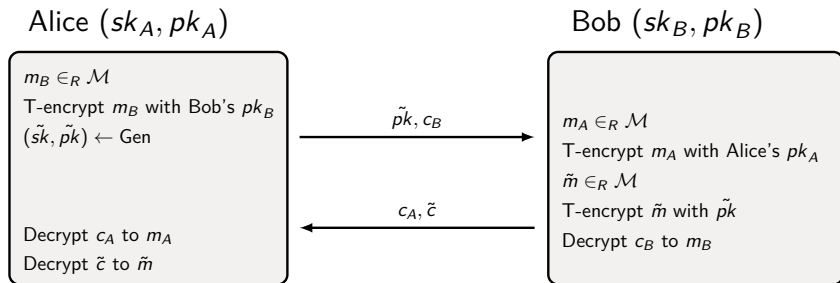
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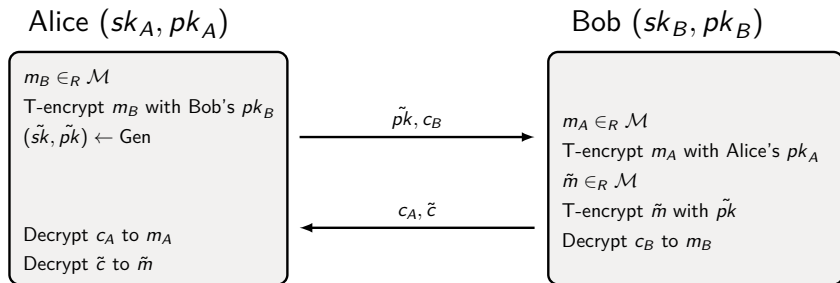


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With **observation**, AKE proof \approx multi-user version of our KEM proof

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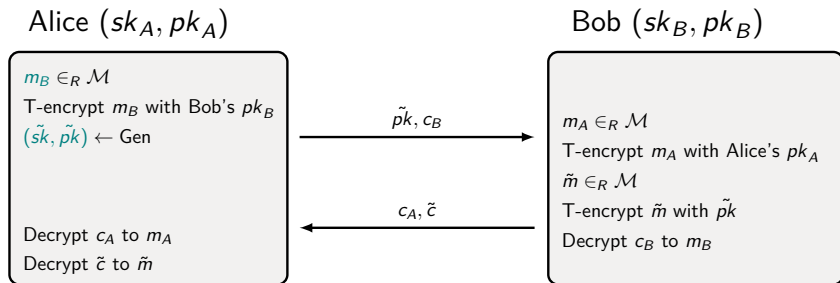


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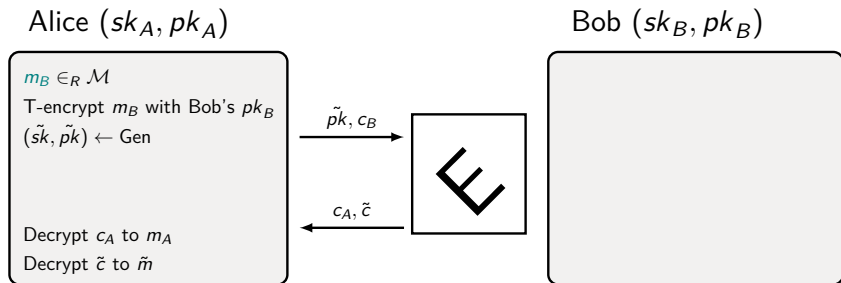
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Alice's state: independent of sk_A

Bob's response (and m_A, \tilde{m}): independent of sk_B

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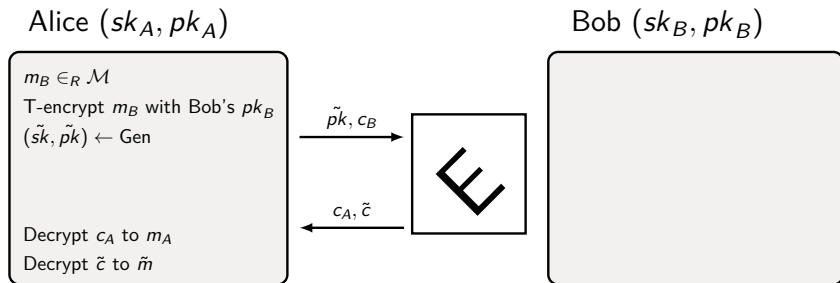
$$K := H(pk_A, pk_B, \tilde{pk}, m_A, m_B, \tilde{m}, c_A, c_B, \tilde{c})$$

Observation: Nontrivial strategy $\rightarrow \mathcal{C}$ only obtains 2 out of (m_i, m_j, \tilde{m})

Exception: Aforementioned 'state reveal attack':

Reveal the state to learn m_B and pretend to be Bob to control m_A, \tilde{m}

Security of our protocol (Intuition)



$$K := H(pk_A, pk_B, \tilde{pk}, m_A, m_B, \tilde{m}, c_A, c_B, \tilde{c})$$

Observation: Nontrivial strategy $\rightarrow \mathcal{A}$ only obtains 2 out of (m_i, m_j, \tilde{m})

Exception: Aforementioned 'state reveal attack':

To succeed, \mathcal{A} has to reveal Alice's session state before time-out

Open questions

Open questions

Active security requires 'worst-case' correctness

→ Can we soften this requirement, generically?

Passive-to-active transformations **starting from KEMs?**

→ Possible applications in AKE and when defining "hybrid" modes

KSSSS20: New quantum extraction technique → Tighter bounds

Can we apply MRM to our proof structure?

→ Tighter bounds for PKE and AKE → Efficiency

References

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